International Journal of Thermal Sciences 49 (2010) 1003–1009

Contents lists available at ScienceDirect

International Journal of Thermal Sciences

journal homepage: www.elsevier.com/locate/ijts

Heat transfer with laminar pulsating flow in a channel or tube in rolling motion

B.H. Yan [a,*], L. Yu [a], Y.H. Yang [b]

[a] Department of Nuclear Science and Engineering, Naval University of Engineering, 717 Jiefang Street, Wuhan 430033, China
[b] School of Nuclear Science and Engineering, Shanghai Jiao Tong University, 800 Dong Chuan Road, Shanghai 200240, China

ARTICLE INFO

Article history:
Received 28 July 2009
Received in revised form
8 January 2010
Accepted 8 January 2010
Available online 16 February 2010

Keywords:
Tube
Parallel-plates channel
Laminar pulsating flow
Nusselt number

ABSTRACT

The heat transfer model of laminar pulsating flow in a channel or tube in rolling motion is established. The correlations of velocity, temperature and Nusselt number are obtained. And the effects of several parameters on Nusselt number are investigated. The results are evaluated with Nield & Kuznetsov's work. It is found that Nield & Kuznetsov's results are not applicable for the laminar pulsating flow in nuclear power systems in ocean environments.

1. Introduction

Forced convection with laminar pulsating flow in a channel or circular tube is a classical problem, and is the subject of hundreds of papers. It appears that the first analytic studies of laminar pulsating forced convection in confined spaces were made by Siegel [1] (for a circular duct) and Siegel and Perlmutter [2] (for a parallel-plates channel). In each case the fluid velocity was approximated by slug flow. Moschandreou [3] studied pulsating flow in a circular tube with uniform constant heat flux on the boundaries. He obtained an expression for fluctuating part of the temperature using a Green's function method. However, Hemida [4] pointed out that the solution of Moschandreou did not satisfy the appropriate differential equation expressing the conservation of thermal energy. Hemida also used a Green's function method to find an analytical expression for the temperature distribution. In order to obtain the Nusselt number they resorted to a numerical calculation. Yu [5] also considered the case of a circular tube with constant heat flux. But he did not present an analytical solution for the bulk temperature nor for the transient Nusselt number. Nield and Kuznetsov [6] have obtained analytical expressions for the velocity, temperature distribution, and transient Nusselt number for the problem of forced convection using a perturbation approach. Numerical studies of flow and heat transfer in a circular tube under pulsating flow condition in the laminar regime were also carried out by Durst [7]. His results indicate that pulsation has no effect on time averaged heat transfer.

As to the nuclear power systems operating in ocean environments which mainly include rolling and heaving motions [8], the coolant within them oscillates periodically. This kind of pulsating flow is similar to that discussed in [1–7]. Some researchers have investigated the pulsating flow in nuclear power systems in ocean environments. A single-phase natural circulation experiment was carried out by Murata [9] for analyzing the effect of rolling motion on the thermal hydraulic characteristic of reactor. Yan [10,11] also investigated the effect of rolling motion on the flowing characteristics of the coolant in a passive residual heat removal system. To the author's knowledge, the effect of rolling motion on the laminar pulsating flow had never been addressed theoretically. In this paper, the heat transfer model of laminar pulsating flow in a tube in rolling motion is established. The effects of several parameters on Nusselt number are investigated. And in the end, Nield & Kuznetsov's results are evaluated with the results in this paper.

2. Circular tubes

Assuming the laminar flow is fully developed, and the Navier–Stokes equation can be written as

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + v\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) \tag{2.1}$$

where u is velocity, ρ is flow density, p means pressure and v denotes kinematic viscosity. x denotes the axial coordinate, which

* Corresponding author. Tel.: +86 27 83443361.
E-mail address: yanbh1986@163.com (B.H. Yan).

1290-0729/$ – see front matter
doi:10.1016/j.ijthermalsci.2010.01.011

Nomenclature

General symbols

d	reference length (m)
h	heat transfer coefficient (W/m^2 K)
K	temporary variable
k	fluid thermal conductivity (W/m K)
L	characteristics length
Nu	Nusselt number
n	frequency (Hz)
p	pressure (Pa)
Pr	Prandtl number
R	tube radius (m)
r	radius (m)
Re	Reynolds number
T	period (s), temperature (°C)
t	time (s)
u	velocity (m/s)
x, y	coordinate (m)

Greek letters

ρ	density (kg/m^3)
υ	kinematic viscosity (m^2/s)
Ω	temporary variables
ω	angle velocity (rad/s)

Superscripts

*	dimensionless variables

Subscripts

0	reference
1, 2	subcomponent
in	inlet
m	maximum, bulk
r	rolling
o	oscillating part
w	wall

is overlapped with the tube axis, and r is the radial distance from the tube axis. The transient pressure drop in a tube can be expressed as

$$-\frac{1}{\rho}\frac{\partial p}{\partial x} = a_r - a_o\cos\frac{2\pi t}{T_r} \tag{2.2}$$

where $a_o = n^2\theta_m L/2$. L and n are the characteristic length of the system and rolling frequency, respectively. θ_m is rolling amplitude and T_r is rolling period.

In order to transform the above equations into a dimensionless form, the following dimensionless parameters are introduced.

$$u^* = u/u_m,\ r^* = r/R,\ x^* = x/(R\mathrm{PrRe}),\ t^* = ta/R^2,\ \rho^* = \rho/\rho_0,\ p^* = p/\rho_0 u_m^2 \tag{2.3}$$

where

$$u_m = R^2 a_r/4\upsilon,\ a = k/\rho_0 C_p \tag{2.4}$$

u_m is the time averaged velocity in the centerline. R is tube radius. Pr and Re denote Prandtl and Reynolds number, respectively. ρ_0 is reference density. k is fluid thermal conductivity and C_p denotes fluid specific heat. The superscript * denotes dimensionless parameter.

The dimensionless N–S equation can be written as:

$$\frac{\partial u^*}{\partial t^*} = -\frac{1}{\rho^*}\frac{\partial p^*}{\partial x^*} + \mathrm{Pr}\left(\frac{\partial^2 u^*}{\partial r^{*2}} + \frac{1}{r^*}\frac{\partial u^*}{\partial r^*}\right) \tag{2.5}$$

where

$$-\frac{1}{\rho^*}\frac{\partial p^*}{\partial x^*} = 4\mathrm{Pr} - \frac{a_o R\mathrm{PrRe}}{u_m^2}\cos\frac{R^2 nt^*}{a} \tag{2.6}$$

Separating the velocity into a steady term and an oscillating term

$$u^*(r^*,t^*) = u_1^*(r^*) + u_2^*(r^*,t^*) \tag{2.7}$$

The equations of $u_1^*(r^*)$ and $u_2^*(r^*,t^*)$ can be expressed as

$$\left(\frac{\partial^2 u_1^*}{\partial r^{*2}} + \frac{1}{r^*}\frac{\partial u_1^*}{\partial r^*}\right) + 4 = 0 \tag{2.8}$$

$$\frac{\partial u_2^*}{\partial t^*} = \mathrm{Pr}\left(\frac{\partial^2 u_2^*}{\partial r^*} + \frac{1}{r^*}\frac{\partial u_2^*}{\partial r^*}\right) - \frac{a_o R\mathrm{PrRe}}{u_m^2}\cos\frac{R^2 nt^*}{a} \tag{2.9}$$

The solution of Eq. (2.8) is a parabola, which can be written as

$$\left(u_1^*(r^*) = 1 - r^{*2}\right) \tag{2.10}$$

According to Eq. (2.9), $u_2^*(r^*,t^*)$ can be solved as:

$$u_2^*(r^*,t^*) = -\frac{K_1}{i\Omega_1}e^{i\Omega_1 t^*}\left[1 - \frac{J_0\left(\sqrt{-i\Omega_1/\mathrm{Pr}}r^*\right)}{J_0\left(\sqrt{-i\Omega_1/\mathrm{Pr}}\right)}\right] \tag{2.11}$$

where

$$\Omega_1 = R^2 n/a,\ K_1 = a_o R\mathrm{PrRe}/u_m^2 \tag{2.12}$$

Substituting Eqs. (2.10) and (2.11) into (2.7) yields

$$u^*(r^*,t^*) = (1 - r^{*2}) - \frac{K_1}{i\Omega_1}e^{i\Omega_1 t^*}\left[1 - \frac{J_0\left(\sqrt{-i\Omega_1/\mathrm{Pr}}r^*\right)}{J_0\left(\sqrt{-i\Omega_1/\mathrm{Pr}}\right)}\right] \tag{2.13}$$

Eq. (2.13) is the dimensionless velocity correlation of incompressible laminar pulsating flow in tubes in rolling motion. It should be noted that Eq. (2.13) neglects the influence of entrance region and it is only applicable for the fully developed flow. The first part of Eq. (2.13) is the average velocity and the second part, the time averaged value of which is zero, denotes the oscillation of velocity against time. It seems that the dimensionless velocity in rolling motion is a function of Prandtl number. But it can be found that $i\Omega_1/\mathrm{Pr} = id^2 n/\upsilon$, and the Prandtl number within $K_1/i\Omega_1$ can be reduced. Both $i\Omega_1/\mathrm{Pr}$ and $K_1/i\Omega_1$ have nothing to do with Prandtl number. So, the velocity is not a function of Prandtl number, but a function of fluid viscosity.

The energy equation of incompressible laminar flow can be written as:

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} = a\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right) \tag{2.14}$$

Assuming

$$T^* = (T - T_w)/(T_m - T_w) \tag{2.15}$$

where T_w and T_m are the wall temperature and bulk temperature, respectively. The uniform wall temperature are available in nuclear systems, like reactor and heat exchangers.

According to Eqs. (2.7) and (2.15), the dimensionless energy equation can be expressed as:

$$\frac{\partial T^*}{\partial t^*}+u^*\frac{\partial T^*}{\partial x^*}=\frac{\partial^2 T^*}{\partial r^{*2}}+\frac{1}{r^*}\frac{\partial T^*}{\partial r^*} \tag{2.16}$$

The boundary conditions of Eq. (2.16) are

$$\begin{cases} T^*(x^*,1,t^*)=0 \\ T^*(0,r^*,t^*)=1 \\ \partial T^*(x^*,0,t^*)/\partial r^*=0 \end{cases} \tag{2.17}$$

The dimensionless temperature can be separated into two parts:

$$T^*(x^*,r^*,t^*)=T_1^*(x^*,r^*)+T_2^*(r^*,t^*) \tag{2.18}$$

The equation of $T_1^*(x^*,r^*)$ can be expressed as

$$u_1^*\frac{\partial T_1^*}{\partial x^*}=\frac{\partial^2 T_1^*}{\partial r^{*2}}+\frac{1}{r^*}\frac{\partial T_1^*}{\partial r^*} \tag{2.19}$$

with boundary conditions

$$\begin{cases} T_1^*(x^*,1)=0 \\ T_1^*(0,r^*)=1 \\ \partial T_1^*(x^*,0)/\partial r^*=0 \end{cases} \tag{2.20}$$

And the equation of $T_2^*(r^*,t^*)$ can be expressed as

$$\frac{\partial T_2^*}{\partial t^*}+u_2^*\frac{\partial T_1^*}{\partial x^*}=\frac{\partial^2 T_2^*}{\partial r^{*2}}+\frac{1}{r^*}\frac{\partial T_2^*}{\partial r^*} \tag{2.21}$$

with boundary conditions

$$\begin{cases} T_2^*(1,t^*)=0 \\ \partial T_2^*(0,t^*)/\partial r^*=0 \end{cases} \tag{2.22}$$

Sellars [12] has solved Eq. (2.19) and got a correlation for the steady-state temperature. In this paper, Sellars' method is introduced. Substituting Eq. (2.10) into (2.19) yields

$$\frac{\partial T_1^*}{\partial x^*}=\frac{1}{1-r^{*2}}\frac{1}{r^*}\frac{\partial}{\partial r^*}\left(r^*\frac{\partial T_1^*}{\partial r^*}\right) \tag{2.23}$$

The solution of Eq. (2.23) can be expressed as:

$$T_1^*(x^*,r^*)=\sum_{m=0}^{\infty}B_mR_m(r^*)e^{-\gamma_m^2x^*} \tag{2.24}$$

where

$$B_m=(-1)^m\frac{2\cdot 6^{2/3}\Gamma(2/3)}{\pi}\gamma_m^{-2/3} \quad m=0,1,2,\ \ldots \tag{2.25}$$

The equations for $R_m(r^*)$ are therefore for small r^* (center of pipe)

$$R_m(r^*)=J_0(\gamma_m r^*) \tag{2.26}$$

for medium r^* $(0<r^*<1)$

$$R_m(r^*)=\sqrt{\frac{2}{\pi\gamma_m r^*}}\frac{\cos\left[\frac{\gamma_m}{2}\left(r^*\sqrt{1-r^{*2}}+\arcsin r^*\right)-\frac{\pi}{4}\right]}{(1-r^{*2})^{1/4}} \tag{2.27}$$

and for big r^* (near the wall)

$$R_m(r^*)=\sqrt{\frac{2(1-r^*)}{3}}(-1)^mJ_{1/3}\left[\frac{\sqrt{8}\gamma_m}{3}(1-r^*)^{3/2}\right] \tag{2.28}$$

$$\gamma_m=4m+8/3 \quad m=0,1,2,\ldots \tag{2.29}$$

The form of solution is rather complicated for calculation purposes but it is possible to deduce from it that $T_1^*(x^*,r^*)$ becomes a linear function of x^* downstream. More accurately, as x^* becomes large,

$$\partial T_1^*/\partial x^* \to 4.0 \tag{2.30}$$

In fact to a good degree of approximation the condition applies for $x^*>1.0$.

This result makes it possible to seek a solution of Eq. (2.21) for $T_2^*(r^*,t^*)$ in the form

$$T_2^*(r^*,t^*)=4f(r^*)e^{i\Omega_1t^*} \tag{2.31}$$

Substituting Eq. (2.31) into Eq. (2.21) yields

$$\frac{d^2f}{dr^*}+\frac{1}{r^*}\frac{df}{dr^*}-i\Omega_1f=g(r^*) \tag{2.32}$$

where

$$g(r^*)=-\frac{K_1}{i\Omega_1}\left[1-\frac{J_0(\sqrt{-i\Omega_1/\mathrm{Pr}}r^*)}{J_0(\sqrt{-i\Omega_1/\mathrm{Pr}})}\right] \tag{2.33}$$

Eq. (2.32) can be solved for $f(r^*)$ to get

$$f(r^*)=C(r^*)J_0(\sqrt{-i\Omega_1}r^*) \tag{2.34}$$

And $C(r^*)$ can be written as:

$$C(r^*)=\int_1^{r^*}\int_0^{r^*}g(r^*)\exp\int_1^{r^*}\left[\frac{-2\sqrt{-i\Omega_1}J_1\left(\sqrt{-i\Omega_1}r^*\right)}{J_0\left(\sqrt{-i\Omega_1}r^*\right)}+\frac{1}{r^*}\right]dr^* \times dr^*\exp\left\{\int_1^{r^*}-\left[\frac{-2\sqrt{-i\Omega_1}J_1\left(\sqrt{-i\Omega_1}r^*\right)}{J_0\left(\sqrt{-i\Omega_1}r^*\right)}+\frac{1}{r^*}\right]dr^*\right\}dr^* \tag{2.35}$$

In the end, the dimensionless temperature of laminar pulsating flow can be expressed as:

$$T^*(x^*,r^*,t^*)=\sum_{m=0}^{\infty}B_mR_m(r^*)e^{-\gamma_m^2x^*}+4C(r^*)J_0(\sqrt{-i\Omega_1}r^*)e^{i\Omega_1t^*} \tag{2.36}$$

where the expressions of B_m, $R_m(r^*)$ and $C(r^*)$ are listed in Eqs. (2.25)–(2.29), and (2.35).

The transient Nusselt number can be expressed as:

$$\mathrm{Nu}=2Rh/k \tag{2.37}$$

According to Eq. (2.28) and (2.36), the transient Nusselt number can be solved as:

$$Nu=\sum_{m=0}^{\infty}\frac{3.3734}{\gamma_m^{1/3}}e^{-\gamma_m^2x^*}+8\int_0^1 g(r^*)r^*\left[\frac{J_0(\sqrt{-i\Omega_1}r^*)}{J_0(\sqrt{-i\Omega_1})}\right]^2dr^*e^{i\Omega_1t^*} \tag{2.38}$$

3. Parallel-plates channel

The analysis for the case of a parallel-plates channel follows closely that for a circular tube, and so for brevity we just list the major changes. In this coordinate, x denotes the flowing direction, and y is the distance to x axis. The dimensionless N–S equation can be written as:

$$\frac{\partial u^*}{\partial t^*} = -\frac{1}{\rho^*}\frac{\partial p^*}{\partial x^*} + \mathrm{Pr}\frac{\partial^2 u^*}{\partial y^{*2}} \tag{3.1}$$

where the reference length d is half of the distance between the plates, the other dimensionless parameters are the same with that of Eq. (2.3). And the dimensionless velocity can be solved as

$$u^*(y^*,t^*) = 1 - y^{*2} - \frac{K_2}{i\Omega_2}e^{i\Omega_2 t^*}\left[1 - \frac{ch\left(\sqrt{i\Omega_2/\mathrm{Pr}}y^*\right)}{ch\left(\sqrt{i\Omega_2/\mathrm{Pr}}\right)}\right] \tag{3.2}$$

where

$$\Omega_2 = d^2 n/a,\ \ K_2 = a_o d\mathrm{PrRe}/u_m^2 \tag{3.3}$$

Eq. (3.2) is the dimensionless velocity correlation of incompressible laminar flow in parallel-plates channels in rolling motion. It should be noted that Eq. (3.2) neglects the effect of entrance region and it is only applicable for the fully developed flow.

The dimensionless energy equation can be expressed as:

$$\frac{\partial T^*}{\partial t^*} + u^*\frac{\partial T^*}{\partial x^*} = \frac{\partial^2 T^*}{\partial y^{*2}} \tag{3.4}$$

The boundary conditions of Eq. (3.4) are

$$\begin{cases} T^*(x^*,1,t^*) = 0 \\ T^*(0,y^*,t^*) = 1 \\ \partial T^*(x^*,0,t^*)/\partial y^* = 0 \end{cases} \tag{3.5}$$

The dimensionless temperature can also be separated into two parts:

$$T^*(x^*,y^*,t^*) = T_1^*(x^*,y^*) + T_2^*(y^*,t^*) \tag{3.6}$$

The solution of $T_1^*(x^*,y^*)$ can be expressed as:

$$T_1^*(x^*,y^*) = \sum_{m=0}^{\infty} B_m Y_m(y^*)e^{-\gamma_m^2 x^*} \tag{3.7}$$

where

$$B_m = (-1)^{m+1}\frac{2^{1/4}6^{4/3}\Gamma(2/3)}{\pi^{3/2}}\gamma_m^{-7/6} \quad m = 0,1,2,\ \ldots \tag{3.8}$$

for small y^* (center of channel)

$$Y_m(y^*) = \cos(\gamma_m y^*) \tag{3.9}$$

for medium y^* $(0 < y^* < 1)$

$$Y_m(y^*) = \frac{\cos\gamma_m\left[\frac{\arcsin y^*}{2} + \frac{\sin(2\arcsin y^*)}{4}\right]}{(1-y^{*2})^{1/4}} \tag{3.10}$$

and for big y^* (near the wall)

$$Y_m(y^*) = (-1)^m\left(\frac{2}{9}\right)^{1/4}\sqrt{\pi\gamma_m(1-y^*)}J_{1/3}\left[\frac{\sqrt{8}\gamma_m}{3}(1-y^*)^{3/2}\right] \tag{3.11}$$

$$\gamma_m = 4m + 1/3 \quad m = 0,1,2,\ \ldots \tag{3.12}$$

And the solution of $T_2^*(y^*,t^*)$ can be expressed as:

$$T_2^*(y^*,t^*) = \frac{K_2}{\Omega_2^2}\left[-1 - \frac{\mathrm{Pr}ch\left(\sqrt{i\Omega_2/\mathrm{Pr}}y^*\right)}{(1-\mathrm{Pr})ch\left(\sqrt{i\Omega_2/\mathrm{Pr}}\right)} + \frac{ch\left(\sqrt{i\Omega_2}y^*\right)}{(1-\mathrm{Pr})ch\left(\sqrt{i\Omega_2}\right)} - \frac{ch\left[\sqrt{i\Omega_2}(1-y^*)\right]}{\sqrt{i\Omega_2}ch\left(\sqrt{i\Omega_2}\right)}\right]e^{i\Omega_2 t^*} \tag{3.13}$$

With Eqs. (3.6), (3.7) and (3.13), the expression of $T^*(x^*,y^*,t^*)$ can be written as:

$$T^*(x^*,y^*,t^*) = \sum_{m=0}^{\infty} B_m Y_m(y^*)e^{-\gamma_m^2 x^*} + \frac{K_2}{\Omega_2^2}\left[-1 - \frac{\mathrm{Pr}ch\left(\sqrt{i\Omega_2/\mathrm{Pr}}y^*\right)}{(1-\mathrm{Pr})ch\left(\sqrt{i\Omega_2/\mathrm{Pr}}\right)} + \frac{ch\left(\sqrt{i\Omega_2}y^*\right)}{(1-\mathrm{Pr})ch\left(\sqrt{i\Omega_2}\right)} - \frac{ch\left[\sqrt{i\Omega_2}(1-y^*)\right]}{\sqrt{i\Omega_2}ch\left(\sqrt{i\Omega_2}\right)}\right]e^{i\Omega_2 t^*} \tag{3.14}$$

Eq. (3.14) is the dimensionless temperature correlation of incompressible laminar pulsating flow in parallel-plates channels in rolling motion.

The transient and average Nusselt number can be expressed as:

$$\mathrm{Nu} = 2dh/k \tag{3.15}$$

Substituting Eq. (3.14) into (3.15) yields:

$$\mathrm{Nu} = \sum_{m=0}^{\infty}\frac{4\sqrt{3}\Gamma(2/3)}{\pi\Gamma(4/3)\gamma_m^{1/3}}e^{-\gamma_m^2 x^*} + \frac{2K_2}{\Omega_2^2}\left[-\frac{\mathrm{Pr}\sqrt{i\Omega_2/\mathrm{Pr}}th\left(\sqrt{i\Omega_2/\mathrm{Pr}}\right)}{1-\mathrm{Pr}} + \frac{\sqrt{i\Omega_2}th\left(\sqrt{i\Omega_2}\right)}{1-\mathrm{Pr}}\right]e^{i\Omega_2 t^*} \tag{3.16}$$

Eq. (3.16) is the correlation of Nusselt number of incompressible laminar flow in parallel-plates channels in rolling motion.

4. Results and discussion

4.1. Comparison

Nield and Kuznetsov [6] have analyzed the heat transfer characteristics of laminar pulsating flow using a perturbation method. So, Nield & Kuznetsov's results are compared with the Nusselt numbers in the present paper. Take the water (20 °C, 0.1 MPa) as an example, assuming $R = 0.05$ m , $d = 0.05$ m, $u_m = 0.01$ m/s. Yan [13] has proved that, as to the nuclear power system, the characteristics length L is usually between 0.1 m and 1.0 m. So we set $L = 0.1$ m. The comparison results of Nusselt numbers with different rolling periods and amplitudes are listed in Tables 1 and 2.

It can be seen from Tables 1 and 2 that the difference between Nield & Kuznetsov's results and the results in the present paper is significant. In Nield & Kuznetsov's analysis, the high order term of perturbation is omitted. So, the precision of his results is determined by the magnitude of perturbation. If the perturbation is big enough, Nield & Kuznetsov's results are not applicable. As to the

Table 1
Comparison of Nusselt number of a circular tube (degree means amplitude).

		5°	8°	10°	12°	15°	20°	25°
10 s	Present work	0.0016	0.0026	0.0033	0.0039	0.0049	0.0065	0.0082
	Nield & Kuznetsov's	0.5724	0.9159	1.1448	1.3738	1.7173	2.2897	2.8621
20 s	Present work	0.0012	0.0019	0.0023	0.0028	0.0035	0.0047	0.0058
	Nield & Kuznetsov's	0.2710	0.4336	0.5421	0.6505	0.8131	1.0841	1.3551

Table 2
Comparison of Nusselt number of a parallel-plates channel (degree means amplitude).

		5°	8°	10°	12°	15°	20°	25°
10 s	Present work	0.0014	0.0023	0.0029	0.0034	0.0043	0.0057	0.0072
	Nield & Kuznetsov's	0.2701	0.4321	0.5401	0.6482	0.8102	1.0803	1.3503
20 s	Present work	0.0010	0.0016	0.0020	0.0024	0.0030	0.0041	0.0051
	Nield & Kuznetsov's	0.1295	0.2072	0.2590	0.3108	0.3884	0.5179	0.6474

nuclear power systems, the ratio of amplitude oscillating pressure and average pressure can be expressed as:

$$\varepsilon = \left|\frac{a_o}{a_r}\right| = \frac{\pi^2 L\theta_m R^2}{2\upsilon u_m T_r^2} \tag{4.1}$$

As to the parameters analyzed above, $\varepsilon \approx 215.3$. In this case, the role of high order term of perturbation is significant, which cannot be omitted. So Nield & Kuznetsov's results are much bigger than that in this paper. It can be concluded that Nield & Kuznetsov's results is applicable for small perturbation. But as to the nuclear power system, the perturbation is usually very big, which cannot be omitted, and the result in this paper is a supplement to Nield & Kuznetsov's results.

Tan [14] has investigated the heat transfer characteristics of single-phase flow in rolling motion experimentally. His results are compared with the results deduced in this paper. The comparing results are listed in Table 3.

It is indicated in Table 1 that the discrepancy between the present results and Tan's results is limited. The smaller the rolling amplitude is, the minor the discrepancy is.

4.2. Circular tubes

The variation of the oscillating amplitude of transient Nusselt number with Prandtl number is investigated, which is shown in Fig. 1. It is shown that the oscillating amplitude of transient Nusselt number increases with the increase of Prandtl number. There is an approximate linear relation between them. It is indicated in Fig. 2 that the variation of initial phase of Nusselt number with rolling frequency is very limited. It can be considered to be a constant, about 35 °.

It is indicated in Fig. 3 that if the rolling period is small, the oscillating amplitude of Nusselt number is very big and decreases quickly with the increase of rolling period. But the oscillating amplitude of Nusselt number can be considered to be a constant if the rolling period is more than 20 s. Fig. 3 also indicates that there is an approximate linear relation between the oscillating amplitude of Nusselt number and rolling frequency. In ocean environment, the rolling frequency is usually less than 1 Hz [8]. So the discussion for higher frequency is not necessary. And the linear relation is available for ocean environment.

4.3. Parallel-plates channel

The variation of the oscillating amplitude of transient Nusselt number in a parallel-plates channel with Prandtl number is investigated, which is shown in Fig. 4. It is indicated that the oscillating amplitude of transient Nusselt number increases with the increase of Prandtl number. There is an approximate linear relation between them.

It is indicated in Fig. 5 that the variations of oscillating amplitude of Nusselt number in parallel-plates channels with rolling period and frequency are similar with that in circular tubes. Fig. 5 also indicates that there is an approximate linear relation between the oscillating amplitude of Nusselt number and rolling frequency in parallel-plates channel.

In Nuclear power systems, $\sqrt{i\Omega_2}$ and $\sqrt{i\Omega_2/\mathrm{Pr}}$ are usually very big, and $th(\sqrt{i\Omega_2})$ and $th(\sqrt{i\Omega_2/\mathrm{Pr}})$ are next to 1. Then Eq. (3.16) can be simplified as:

$$\mathrm{Nu} = \sum_{m=0}^{\infty}\frac{3.3442}{\gamma_m^{1/3}}e^{-\gamma_m^2 x^*} + \frac{2K_2}{\Omega_2^2}\left[-\frac{\mathrm{Pr}\sqrt{i\Omega_2/\mathrm{Pr}}}{1-\mathrm{Pr}} + \frac{\sqrt{i\Omega_2}}{1-\mathrm{Pr}}\right]e^{i\Omega_2 t^*} \tag{4.2}$$

Eq. (4.2) is not a good approximation to the solution as $\mathrm{Pr} \to 1$, since it has a singularity there. So Eq. (4.2) is rewritten as:

$$\mathrm{Nu} = \sum_{m=0}^{\infty}\frac{3.3442}{\gamma_m^{1/3}}e^{-\gamma_m^2 x^*} + \frac{2K_2}{\Omega_2^2}\frac{\sqrt{i\Omega_2}}{1+\sqrt{\mathrm{Pr}}}e^{i\Omega_2 t^*} \tag{4.3}$$

It is indicated in Eq. (4.3) that the initial phase of transient Nusselt number is a function of $\sqrt{i\Omega_2}$. And it can be easily proved that the real part of $\sqrt{i\Omega_2}$ is equal to its imaginary part. So the initial

Table 3
Comparison with Tan's results (degree means amplitude).

		5°	8°	10°	12°	15°	20°	25°
10 s	Present work	0.0016	0.0026	0.0033	0.0039	0.0049	0.0065	0.0082
	Tan's results	0.0019	0.0026	0.0030	0.0033	0.0042	0.0058	0.0075
20 s	Present work	0.0012	0.0019	0.0023	0.0028	0.0035	0.0047	0.0058
	Tan's results	0.0015	0.0018	0.0022	0.0024	0.0028	0.0039	0.0050

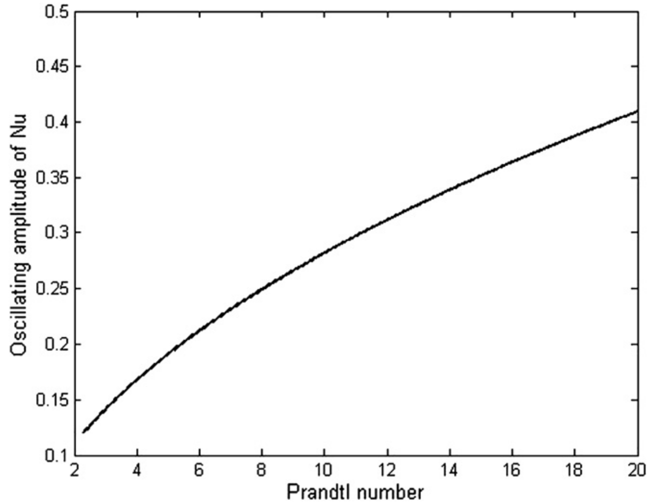

Fig. 1. Variation of oscillating amplitude of Nu with Pr.

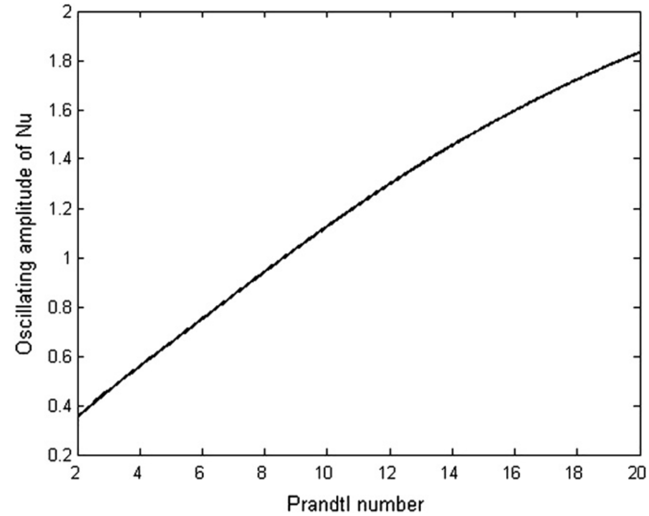

Fig. 4. Variation of oscillating amplitude of Nu with Pr.

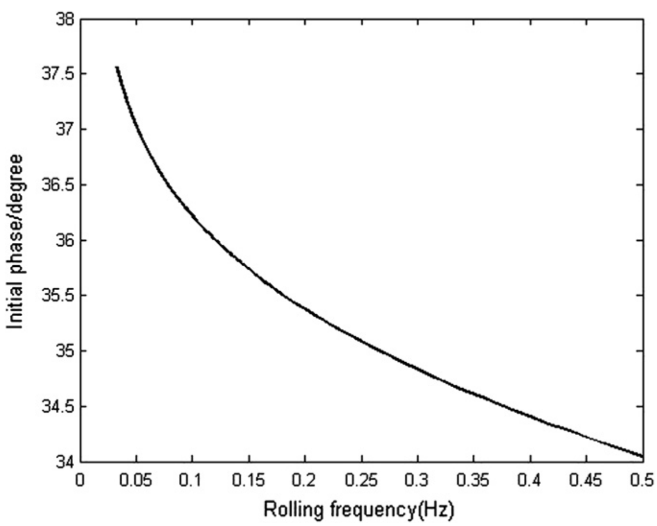

Fig. 2. Variation of initial phase of Nu with rolling frequency.

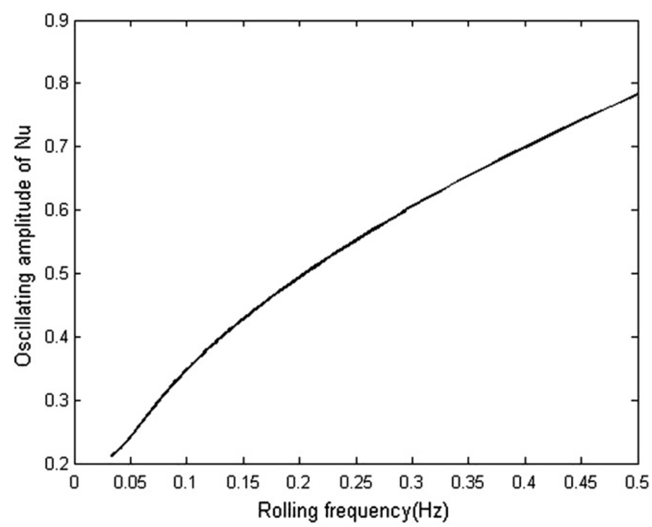

Fig. 5. Variation of oscillating amplitude of Nu with rolling frequency.

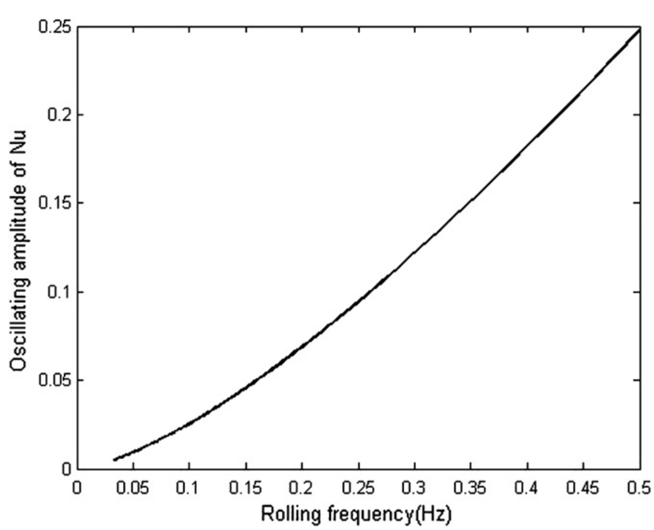

Fig. 3. Variation of oscillating amplitude of Nu with rolling frequency.

phase of the transient Nusselt number is 45 °. So, Eq. (4.3) can be rewritten as:

$$\mathrm{Nu} = \sum_{m=0}^{\infty} \frac{3.3442}{\gamma_m^{1/3}} e^{-\gamma_m^2 x^*} + \frac{2K_2 \cos(nt + \pi/4)}{\Omega_2^{3/2}\left(1 + \sqrt{\mathrm{Pr}}\right)} \tag{4.4}$$

5. Conclusions

The heat transfer characteristic of laminar pulsating flow in a channel or tube in rolling motion is investigated theoretically. The correlations of velocity, temperature and Nusselt number are obtained. And the effects of several parameters on Nusselt number are investigated. The oscillating amplitude of Nusselt number increases with the increase of Prandtl number. There is an approximate linear relation between the oscillating amplitude of Nusselt number and rolling frequency. The initial phase of Nusselt number can be considered to be a constant. Nield & Kuznetsov's results are not applicable for the laminar pulsating flow in nuclear power systems in ocean environments. Because Nield's results are derived from minor disturbance method, while in rolling motion, the disturbance is significant.

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